Mathematics Assignment For Class XII

General Directions For Students :Whatever be the notes provided , everything must be copied in the maths copy and then do the homework in the same copy

Chapter 2: Inverse Trigonometric Function (Part -3)

Topic : Properties of Inverse Trigonometric Function

Please note: Properties of inverse trigonometric functions have been discussed in the video link provided to you along with this assignment.

Exercise 2.2 Q.5. Prove that: $\sin\left(\cot^{-1}\left(\cos\left(\tan^{-1}x\right)\right)\right)=\sqrt{\frac{x^2+1}{x^2+2}}$

Let $\tan^{-1}x=y \Rightarrow x=\tan y$.

$\therefore\ \sec y=\sqrt{1+\tan^2 y}=\sqrt{1+x^2} \Rightarrow \cos y=\sqrt{\frac{1}{1+x^2}}$

Let $\cot^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=z \Rightarrow \cot z=\frac{1}{\sqrt{1+x^2}}$

$\therefore\ \operatorname{cosec} z=\sqrt{1+\cot^2 z}=\sqrt{1+\frac{1}{1+x^2}}=\sqrt{\frac{2+x^2}{1+x^2}} \Rightarrow \sin z=\sqrt{\frac{x^2+1}{x^2+2}}$

L.H.S.= $\sin\left(\cot^{-1}\left(\cos\left(\tan^{-1}x\right)\right)\right)=\sin\left(\cot^{-1}\left(\cos y\right)\right)=\sin\left(\cot^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$

$=\sin z=\sqrt{\frac{x^2+1}{x^2+2}}$

Q7(ii)Prove $\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}=\tan^{-1}\frac{2}{9}$

solution: $\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}.\frac{1}{13}}\right)$ (Here $\frac{1}{7}\times\frac{1}{13}=\frac{1}{91}<1$)

$=\tan^{-1}\frac{20}{90}=\tan^{-1}\frac{2}{9}$

Q6viii). Prove : $\cot^{-1}1+\cot^{-1}2+\cot^{-1}3=\frac{\pi}{2}$

solution:

L.H.S.= $\cot^{-1}1+\cot^{-1}2+\cot^{-1}3$

$=\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$

$=\tan^{-1}1+\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}\right)$

$=\frac{\pi}{4}+\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)=\frac{\pi}{4}+\tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)=\frac{\pi}{4}+\tan^{-1}(1)$ Here $\left(xy=\frac{1}{2}\times\frac{1}{3}<1\right)$

$=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

Q8.(vii) Prove the following : $\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{2}{\sqrt{5}}=\cot^{-1}\frac{2}{11}$

solution. $\sin^{-1}\frac{4}{5}=\tan^{-1}\frac{\frac{4}{5}}{\sqrt{1-\left(\frac{4}{5}\right)^2}}=\tan^{-1}\frac{\frac{4}{5}}{\frac{3}{5}}=\tan^{-1}\frac{4}{3}$ $\because \sin^{-1}x=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$

and $\cos^{-1}\frac{2}{\sqrt{5}}=\tan^{-1}\frac{\sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2}}{\frac{2}{\sqrt{5}}}=\tan^{-1}\frac{\frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}}=\tan^{-1}\frac{1}{2}$ $\because \cos^{-1}x=\tan^{-1}\frac{\sqrt{1-x^2}}{x}$

Now $\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{2}{\sqrt{5}}=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{2}$

$=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{2}=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{2}}{1-\frac{4}{3}\times\frac{1}{2}}\right)=\tan^{-1}\left(\frac{\frac{11}{6}}{\frac{2}{6}}\right)=\tan^{-1}\frac{11}{2}$ $\left(xy=\frac{4}{3}\times\frac{1}{2}<1\right)$

$=\tan^{-1}\frac{11}{2}=\cot^{-1}\frac{2}{11}$ $\because \tan^{-1}x=\cot^{-1}\frac{1}{x}$

Homework:

Exercise 2.2 Q 7.iii),v), vii), Q.8.iv), vi)Q.10.i), iv)